Economia & Management n.3, 2024
DOI: 10.1485/1120-5032-202403ENG-7

STEFANO MENGOLI

INSIGHTS FOR THE ITALIAN MARKET

VALUING COMPANIES WITH EQUITY BETA

The purpose of this paper is to compare the effects of using different methodologies to empirically estimated equity betas in business valuation. In particular, the effects of using returns calculated at different frequencies, i.e., daily, weekly and monthly, and more or less econometrically reliable cutoffs are compared. In addition, the temporal stability of the estimates is examined by reporting the results obtained at the sectoral level. The aim is to provide useful guidance for practitioners, avoiding the uncritical application of methodologies developed with reference to financial contexts that are quite different from the Italian scenario.

ACCOUNTING AND CONTROL//BETA EQUITIES//BUSINESS VALUATION//ITALIAN MARKET//FINANCIAL SYSTEM

STEFANO MENGOLI
is Full Professor of Corporate Finance at the Alma Mater Studiorum - University of Bologna. He is Director of the Department of Economics and Coordinator of the Master of Science in Business Management. His research interests include the impact of geography and corporate governance on corporate finance.

INTRODUCTION

Valuing a company requires a thorough understanding of the underlying theories of corporate finance. Such expertise is necessary not only to apply the approaches established in the literature with rigor, but also to guide decisions, particularly when information is insufficient. In the latter respect, in an underdeveloped financial market such as Italy's, one of the most challenging steps in valuation is undoubtedly determining the cost of capital. As already mentioned, the theoretical framework for identifying the cost of capital is well established. However, the comparison with reality weakens the theoretical certainties, which are usually developed based on assumptions and models that are far removed from concrete situations. In particular, we should point out that empirical studies on the subject have focused almost exclusively on Anglo-Saxon contexts, which are not terribly similar to ours, for example with regard to the development of financial markets. The number of listed companies used to estimate valuation parameters, or the identification of risk factors capable of explaining business risk and, consequently, the cost of capital: these are just a few examples of the diversity of contextual conditions that we can count. With regard to this last point, it is worth noting that in the US market, equity β is now considered one of many risk factors identified in the literature to measure the cost of capital (see, among others, Fama and French, 1992; 1993; 1996). This contrasts with the Italian context, where such an approach does not seem to be so well established. Consequently, the lack of in-depth study has led to apparent confusion and discrepancies with the economic-financial literature, which mainly refers to other scenarios.

Even in the Anglo-Saxon context, however, in practice β is still the only risk factor considered in most cases. In fact, an influential academic study by Graham and Harvey (2001), reporting the results of a questionnaire administered to CFOs of U.S. companies, shows that 73% of respondents "always" or "almost always" use β of the CAPM as the sole factor for estimating the cost of capital. The study also highlights that, paradoxically, it is large companies that have this preference. Evidence of this type is also found in a similar work by Mengoli and Sandri (2012), which instead deals with Italian companies. Once again, the analysis confirms that β is indispensable in estimating the cost of capital. Yet Italian CFOs seem to be divided between those who determine β independently and those who refer to β calculated by analysts rather than practitioners. In any case, both studies show that this method is the most widely used in practice and is preferred to other more or less complex alternatives.

This is perhaps one reason why numerous studies have focused almost exclusively on trying to better understand β and its possible limitations. To name just a few, Carroll and Wei (1988) demonstrate the need for a relationship that is not limited to the linearity of β, while Chen et al. (1986) suggest using it in relation to other macroeconomic variables. Hollstein et al. (2020), to reevaluate β with respect to factors such as those proposed by Fama and French, show that with intraday data, we can better explain the "size" effect without adding other factors. Therefore, they more or less explicitly advocate an Occam's razor logic.

Some studies analyze how to calculate β correctly. Roman and Terrazza (2018), using French stocks in the CAC40 index and choosing an E-Garch model over a simple OLS method, consider daily data and investigate which period is appropriate to use. An interesting result highlighted by the authors concerns the OLS estimates, which tend to demonstrate greater divergence than the more sophisticated model with high β securities. The issue of the interval effect is specifically addressed by several authors, including Scholes and Williams (1977), Dimson (1979), and Corhay (1991). The latter, for example, focuses on stocks listed on the Brussels Stock Exchange, concluding that it is preferable to avoid using time intervals that are too short for measuring returns when calculating β. In addition, the study presents much more evidence, such as the fact that calculations based on weekly data (i.e., returns calculated from Monday to Monday rather than from Friday to Friday) lead to higher β estimates. Martikainen (1991), who analyzes stocks listed on the Helsinki Stock Exchange, also examines the effect of non-synchronous trading bias and how it explains why βs increase with the frequency of returns grows: indeed, Ðs calculated on daily returns are smaller than those calculated on weekly returns and even smaller than those calculated on monthly returns. Agrrawal, Gilbert, and Harkins (2022) go further and test which βs are more predictive of subsequent returns depending on the frequency of the data used in the estimate, and find that Ðs calculated on a weekly basis emerge as the best performers as far as predicting both subsequent weekly and monthly returns.

Perhaps most interesting, especially in the Italian context, is the need to measure the unsystematic risk component, which Lakonishok and Shapiro (1986) demonstrate is crucial. In this respect, Smith and Smith (2004) go further and estimate the cost of non-diversification in quantitative terms. Using a database of high-tech IPOs to estimate the cost of capital for entrepreneurs and VCs, the authors find that owning an undiversified portfolio is two to four times more costly than estimated using β alone compared to a diversified portfolio. Specifically, in a typical case, the cost of capital would more than triple from 11.4% to 40.0%.

The main objective of this paper is to provide up-to-date evidence the empirical estimation of equity β, with reference to companies operating mainly in the Italian context. The aim is to offer potentially actionable indications for practitioners who need to determine the cost of capital in company valuations. In particular, we highlight the peculiarities and effects of the various methods normally applied to determine equity β to offer guidelines for choices in different situations. In fact, thanks to these insights, the appraiser can avoid making uncritical decisions regarding the methodologies to adopt for the estimation, well aware that extant research may apply to contexts very different from the Italian one.

Specifically, this study focuses on the estimation of equity β, a crucial parameter in the equilibrium model of a financial market. Developed independently by Sharpe (1964), Lintner (1965), and Mossin (1966), equity β is a measure of the systematic risk of a financial asset, i.e., the tendency of the return on an asset to vary as the market changes. Statistically, this parameter is measured by the ratio of the covariance of a stock's return to the market return with the variance of the market return. Thus, equity β is a financial measure that can provide useful insights into business risk; consequently, it forms the basis for determining the cost required by shareholders to finance said risk (see Berk and De Marzo, 2013; Guatri and Bini, 2005). The most direct way to estimate equity β is to regress stock returns against market returns. Once the methodology is defined, however, discrepancies emerge in the choice of the frequency of returns (daily, weekly or monthly) and in the level of estimation quality required for the parameter to be accepted. Therefore, the objective of this paper is to compare the impact of these two dimensions on the valuation of corporate equity.

It is worth noting that one way around this problem might be to use βs provided directly by well-known financial sites such as *Yahoo Finance*, which is perhaps the most famous and widely used. However, while these βs may seem like a convenient alternative, they have several limitations. In particular, to reiterate the risk of using black-box estimates, the βs provided by *Yahoo Finance* are based on monthly estimates over a three-year (36-month) horizon, and factor in closing prices rather than capital-adjusted prices to calculate returns. These considerations already reveal the obvious limitations of this parameter, but going further, consider that these βs are computed using the S&P500 market index as a reference, i.e. a U.S. financial market indicator consisting only of large caps. These arguments seem sufficient to suggest that we should not trust this source, making it all the more necessary to examine calculation methods more closely to avoid relying uncritically on estimates of equity β.

Considering the latest available date, i.e. early March 2024, this paper makes β estimates using the methods illustrated on a sample of listed companies based in Italy. The estimates were then compared to ascertain the economic impact of the different choices on the value of corporate equity using an equity-side approach. The temporal stability of the parameters was also analyzed and, perhaps most importantly, the results were disaggregated to construct subsamples based on the sectoral affiliation of the different firms in the sample.

The paper is structured as follows: The next section describes the dataset and the methods to determine equity β. The third section presents the results, while the fourth section examines the economic implications of the estimation choices on the value of firm equity. Finally, in the fifth and last section, after analyzing the implications of the study, the main conclusions are drawn.

DATA

The sample consists of all stocks listed on the Italian Stock Exchange as of March 2024. The initial dataset, obtained from Eikon Thomson Reuters, counted 460 companies. From these, companies were selected that are domiciled in Italy and have a monthly historical series of stock prices for a period of at least 24 months, reducing the sample to 435 observations. The latter criterion was introduced because it is a parameter recognized in the financial literature for obtaining reliable estimates, especially with respect to stock prices calculated on a monthly basis. To compare the estimates, stock returns were then calculated at daily, weekly, and monthly intervals based on daily prices. These three frequencies are normally applied by appraisers to determine the cost of capital for companies. Specifically, adjusted closing returns calculated from Monday's prices were used for weekly returns, and prices from the last trading day of the month for monthly returns.

To determine equity βs, different estimation periods were considered depending on the frequency of equity returns estimates: a one-year time series for daily returns; a two-year time series for weekly returns; and finally, a five-year time series for monthly returns. Once again, this choice was dictated by what practitioners normally implement when conducting a valuation. The market model used to estimate β is as follows:

$$r_{it} = + \alpha + \beta \, r_{mt} + \varepsilon_{it} \qquad t = 1, \ldots, T$$

where *i* is the i-th firm, *m* is the market index, *ε* is the error symbol and *T* is the period, which differs for daily, weekly and monthly estimates.
To verify the stability of the estimates, from 2015 the time series was extended by repeating the same methodology and calculating the βs of each company in the sample for each month starting in January 2020. However, it is important to reiterate that much of our work focused on the more recent estimation period. Once again, we compared the parameters we obtained.

To estimate β, we constructed a market index by calculating the simple averages of the returns of all securities over the three time periods. As a result, the index is equity weighted. In an influential study, Treynor (2005) elaborates on the clear superiority of this type of index, particularly for minimizing estimation errors. When using a value-weighted index, in fact, these errors are less likely to offset and may even cancel each other out, leading to inaccurate results. In concrete terms, the perspective is that of an investor who gives equal weight to small and large companies, considering size in terms of market capitalization. Using an equity-weighted index also makes it possible to limit the market capitalization bias by giving equal weight to each company in the index. Given these salient characteristics, including lower stock concentration, the choice of this type of calculation is essentially compulsory to obtain reliable estimates.

Next, by regressing the stock returns on the market returns, β was estimated by intentionally using an extremely simple econometric model, namely Ordinary Least Squares (OLS), since this is the most common approach for valuation estimates of companies operating in Italy. Therefore, no model correction was applied, for example, for the outliers, autocorrelation or volatility clusters. Consequently, the statistical significance of the estimates presented below refers to simple OLS models.
Finally, we should add that the models are extremely parsimonious: there are no other explanatory variables or lags of market returns.

Note that the βs of a fixed effects panel model and a random effects panel model were first calculated using daily, weekly, and monthly data. For all models, a beta value of one was obtained, at least to the second decimal place and with high statistical significance. However, the model constants, which were calculated differently in the two analyses, did not significantly deviate from zero. These results seem to be a positive indication of the reliability of our data.

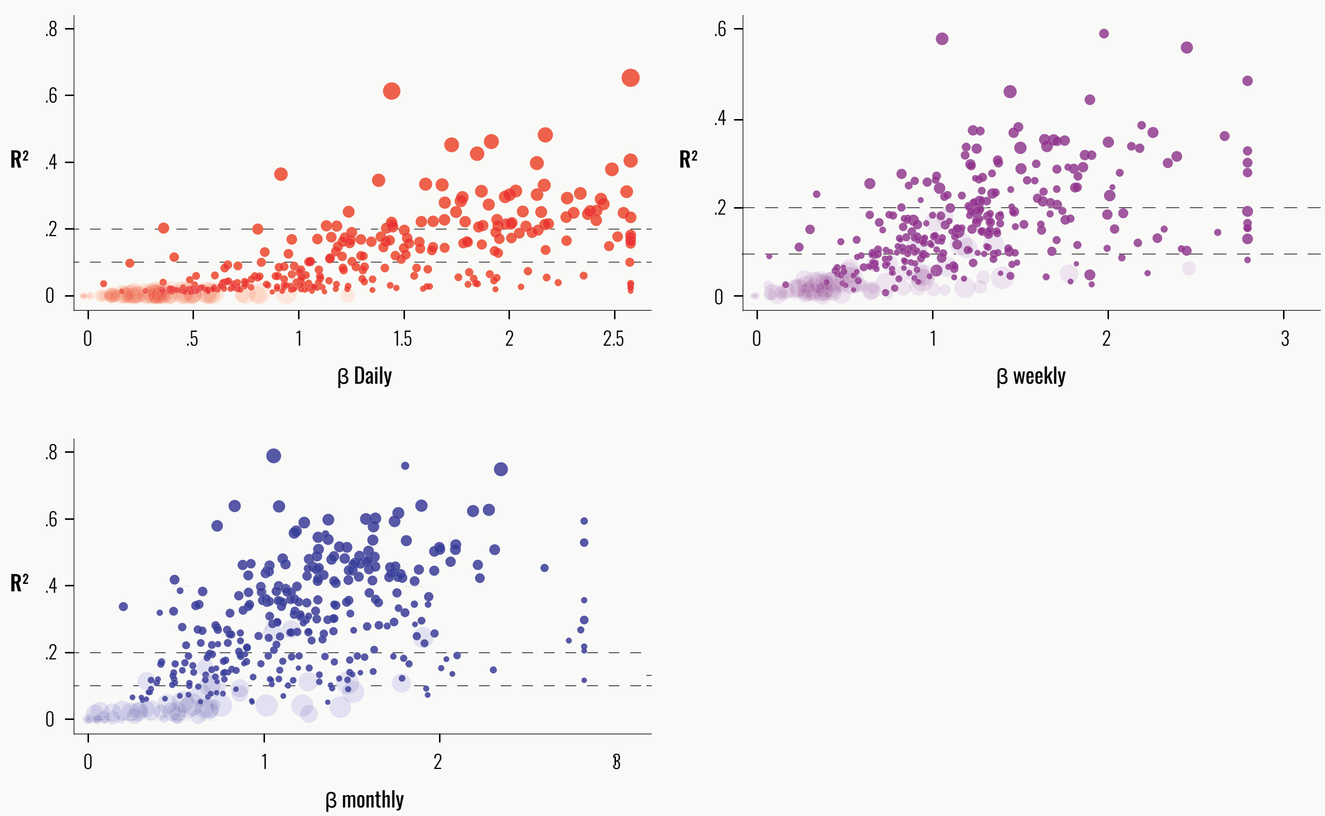
Finally, for each company, and again using Eikon Thomson Reuters, data were collected on the cost of debt and the weight of debt relative to equity based on the most recent balance sheets available. This information also served to calculate the weighted average cost of capital (WACC), and then determine the equity value of a hypothetical firm with ten million euros per year at infinity as its only cash flow. The purpose of this extreme simplification is to provide an indication of the possible effects on value, if any, of decisions made by focusing solely on how equity β is estimated.

RESULTS

Figure 1 shows both the distribution of β estimates for the companies in the sample and the relationship between these βs and the respective coefficients of determination (R^2) obtained from OLS regressions designed to estimate each parameter. Indeed, R^2 is often used by practitioners to provide valuable indications regarding the quality of estimates. As a reminder, this statistical coefficient, with values between zero and one, indicates how much of the change in the dependent variable (in this case stock returns) is explained by the change in the independent variable (in this case market index returns).

Specifically, the scatterplots in Figure 1 show the β and R^2 obtained using daily (first plot, red dots), weekly (second plot, purple dots), and monthly (third plot, blue dots) returns. In each graph, the R^2 cutoffs (10% and 20%) are indicated by the dashed lines; these are typically applied to assess the quality of the estimate and determine whether the parameter is adequate. Finally, we attempted to provide some indication of the statistical significance of the estimate associated with each β parameter. Betas with t-test p-values greater than 10% (not statistically significant) are marked with light shaded dots, while the remainder (statistically

FIGURE 1. DISTRIBUTION OF β AND RESPECTIVE R2 CALCULATED AT THE BEGINNING OF MARCH 2024 ON THE THREE DIFFERENT TIME FRAMES



significant) are marked with colored dots; the size is inversely proportional to the p-value of the estimate. In other words, the larger the dot, the more statistically reliable the estimate.

Analyzing the three scatter plots, we note that the βs are always positive and that there are no stocks that systematically move in the opposite direction of the stock market. On closer inspection, we can see that the econometrically insignificant cases (light shaded dots), i.e. those that should not be considered in a rigorous valuation analysis, tend to cluster at extremely low β values and, of course, on equally low R^2 values close to zero. This initial observation suggests that a non-rigorous valuation, which would consider all βs equally regardless of the quality of the estimate, would lead to an underestimation of equity β and, consequently, to a low assessment of the firm's cost of capital, which then translates into an overestimation of the firm's value.

Comparing the three graphs, it is also clear that as the frequency of returns in determining β increases, so do the coefficients of determination (R^2). This suggests that, whenever possible, it is preferable to opt for weekly or, better still, monthly frequencies rather than daily frequencies. In general, Figure 1 shows that a 10% R^2 cutoff is a simple yet effective solution to discriminate the quality of β estimates, allowing us to select only the ones that are truly econometrically informative.

The last panel of Figure 1 also illustrates the three distributions of βs in the different time frames to confirm our initial findings. Indeed, the frequency distribution of βs computed on a daily basis (red dots) is more even than for weekly (purple dots) or monthly (blue dots) calculations. In fact, the latter tend to look more like a normal distribution, in contrast to the daily data. In all the distributions, however, the mode values tend to cluster around values slightly greater than one.

From the graphical analysis, we turn to Table 1 to examine the descriptive statistics of the three samples. For the 435 companies in the full sample whose βs were calculated using daily, weekly, and monthly frequencies, the average value of βs is around one across all frequencies. Specifically, the average β is 0.98 using daily data, 0.97 using weekly data, and 0.99 using monthly data (column 3).

The average β values are quite similar across the three samples. However, the second column of Table 1, which reports the percentage of statistically significant coefficients with respect to the 10% p-value cutoff associated with the parameter t-test, highlights a greater adequacy of the βs calculated on a monthly basis, with 75% of the estimates being statistically significant. On the other hand, the daily βs are the least adequate in this dimension, as only 70% are significant and therefore statistically

TABLE 1. DESCRIPTIVE STATISTICS OF β BY FREQUENCY OF CALCULATION OF YIELD

	n	pvalue(t-test)>10%	μ	σ	med
Entire sample:					
daily	435	70.34%	0.98	0.79	0.87
weekly	435	71.26%	0.97	0.74	0.96
monthly	435	75.40%	0.99	0.68	1.01
R2>10%:					
daily	128	100.00%	1.75	0.59	1.81
weekly	220	96.82%	1.31	0.69	1.25
monthly	315	94.92%	1.22	0.59	1.19
R2>20%:					
daily	64	100.00%	1.86	0.63	1.98
weekly	94	100.00%	1.45	0.75	1.47
monthly	220	96.82%	1.32	0.59	1.28

acceptable. The information on the mean square deviation (σ) measured for the βs in the three samples suggests that the estimates are more homogeneous for the data with monthly frequency (σ=0.68), registering less dispersion than for the weekly (σ=0.74) and daily (σ=0.79) data, similar to what already emerged in the frequency distributions in Figure 1.

The remainder of Table 1 reports the same statistics with respect to the subsamples obtained using R^2 cutoffs at 10% and 20%. As we can see, the sample size is significantly smaller, particularly with respect to the βs obtained using daily returns: with a 10% cutoff, the sample is reduced from the previous 435 observations to 128, while with a 20% cutoff it decreases further to 64 observations. This represents a roughly sevenfold reduction from the original sample. At the monthly frequency, the decrease is somewhat smaller: from 435 to 220 observations using the 20% cutoff, so the sample is nearly halved. In other words, β estimates made with monthly data have a higher econometric quality.

Worth noting is the obvious consequence of using these cutoffs: the consistent increase of the average βs, in line with what we previously observed in a simple graphical analysis. In fact, non-significant βs tend to be concentrated at much lower values, around zero, which is why when they are factored in, the result is an underestimation of the cost of corporate capital. Again, the monthly frequency least is discriminated by the choice of R^2 cutoff. This initial evidence shows that, also from a methodological point of view, it is indispensable to apply some statistical indicator, such as the coefficient of determination, R^2, to ascertain the quality of the estimates and pinpoint which ones should be considered. The critical consequence of neglecting this aspect is an underestimation of the cost of capital, which in turn results in an overestimation of the value of the company. At the

FIGURE 2. STABILITY OF STOCK β CALCULATED FROM JANUARY 2020 TO MARCH 2024 IN THE THREE DIFFERENT TIME FRAMES

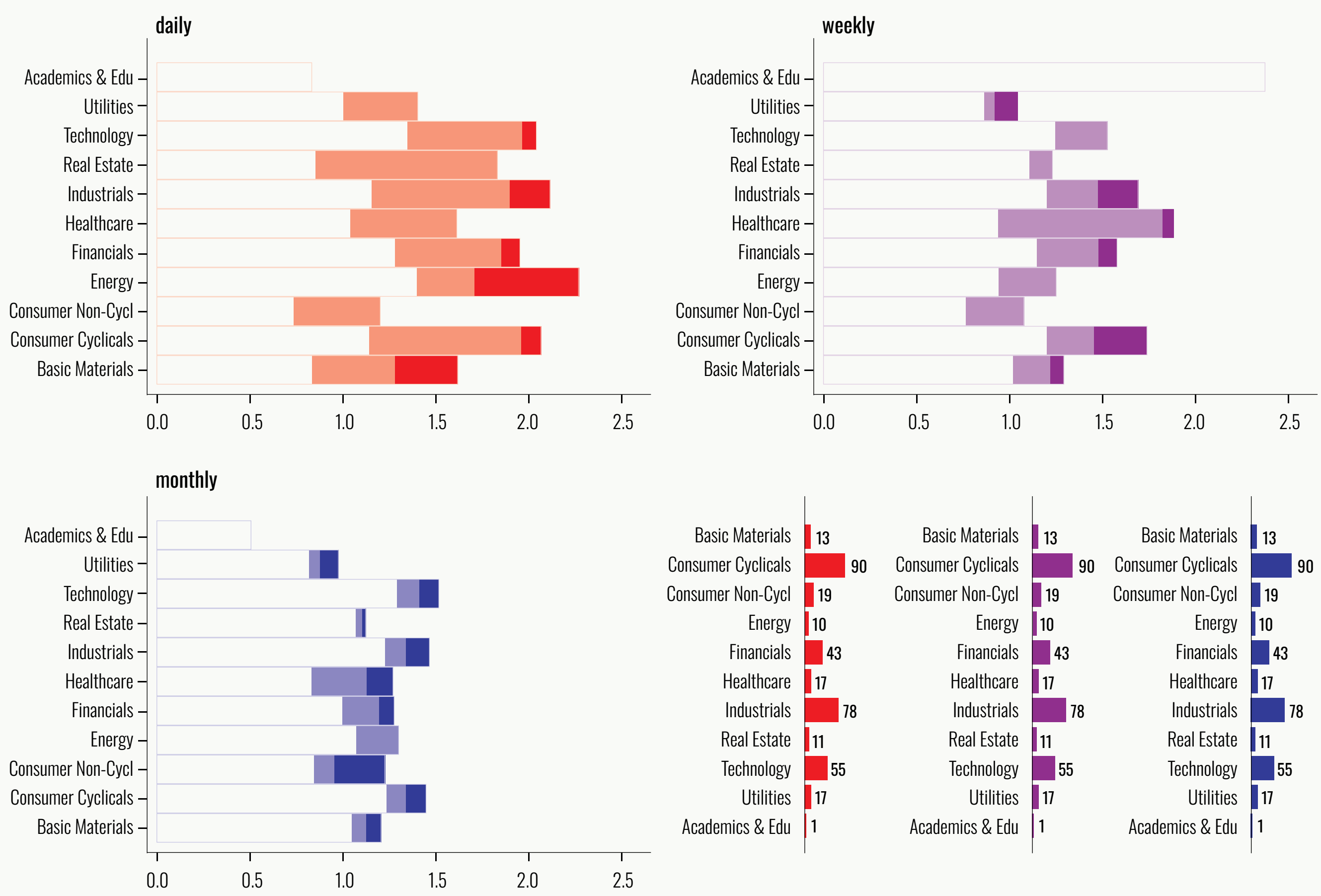
same time, the drastic reduction in the number of companies that can serve as benchmarks for estimates, particularly using daily data, becomes equally evident. In other words, we see the classic dilemma practitioners face every day when valuing an Italian company: they have no choice but to rely on parameters of poor econometric quality as the basis for their estimates.

Since the statistics discussed so far refer only to the sample of parameter estimates from the most recent period (March 2024), Figure 2 provides the results obtained to check the stability of these estimates over time if they were calculated in a different reference period. To do this, βs were recalculated using the same methodology, but placed at the beginning of each month starting in January 2020. In other words, for each month in the window from January 2020 to March 2024, we attempted step into the shoes of a hypothetical appraiser by repeating the exercise we already proposed.

Figure 2 gives the time series of mean (*cross-sectional*) βs for each month of the estimate over the period in question. In particular, the area shown in the graph highlights the 90% confidence interval of each monthly estimate, that is, the range in which 90% of the β values fall, providing a measure of the reliability of the estimates throughout the period. As we can see, the βs remain quite steady over time, even considering the highly uncertain and unstable context of the pandemic and post-pandemic period, which began impacting the financial markets in March 2020.

It is common practice, especially when valuing unlisted companies, to refer not a single comparable

FIGURE 3. SECTOR-AVERAGE β OF RETURN CALCULATIONS ON THE THREE DIFFERENT TIMEFRAMES FOR THE ENTIRE SAMPLE (WHITE), THE SAMPLE WITH R2 > 10% (LIGHT COLOR) AND FINALLY THE SAMPLE WITH R2 > 20% (DARK COLOR)



company but a sample of benchmark companies with similar characteristics (e.g., doing business in the same sector) to calculate the company's riskiness, and the cost of capital. Therefore, an appraiser would typically consider the industry to identify systematic risk. For this reason, as shown in Figure 3 and Table 2 (which contain the same information), the sample companies were classified by industry according to the Refinitiv Business Classification (TRBC). This serves as a standard industry classification to categorize companies at different levels based on similar industries. At the first level, companies are assigned to eleven business sectors: *Basic Materials*, *Consumer Cyclicals*, *Consumer Non-Cyclicals*, *Energy*, *Financials*, *Healthcare*, *Industrials*, *Real Estate*, *Technology Utilities*, *and Academic & Education*. In our study, we limited ourselves to this first level because, even with this breakdown, the *Academic & Education* sector would be excluded once a minimum cutoff is introduced to discriminate the quality of

TABLE 2 DESCRIPTIVE STATISTICS OF β BROKEN DOWN BY SECTOR OF ACTIVITY

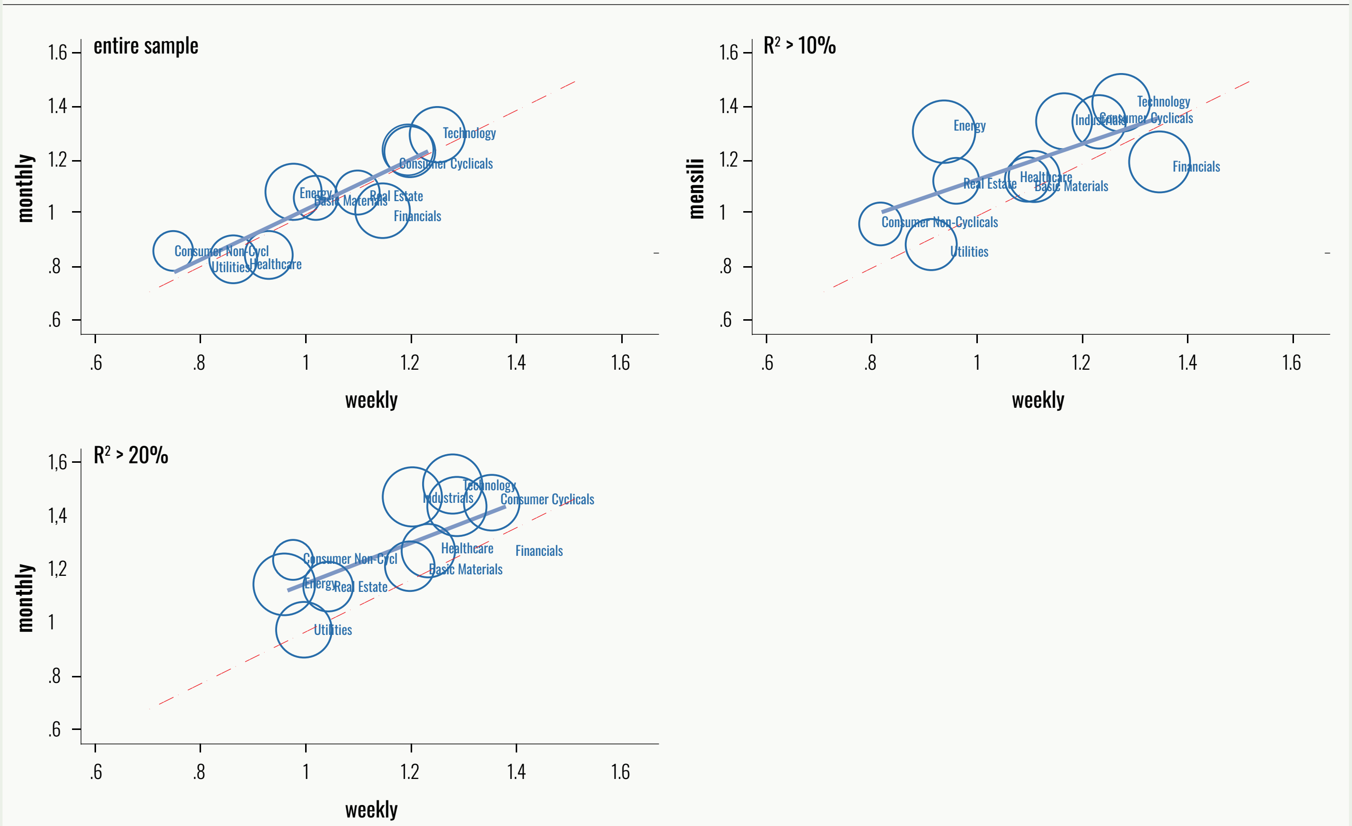
	daily		weekly		monthly	
	N	μ	N	μ	N	μ
			entire sample			
Basic Materials	13	0.84	13	1.01	13	1.05
Consumer Cyclicals	90	1.14	90	1.18	90	1.24
Consumer Non-Cyclicals	19	0.74	19	0.75	19	0.86
Energy	10	1.39	10	0.97	10	1.08
Financials	43	1.28	43	1.13	43	1.00
Healthcare	17	1.04	17	0.92	17	0.84
Industrials	78	1.16	78	1.18	78	1.23
Real Estates	11	0.86	11	1.09	11	1.07
Technology	55	1.34	55	1.23	55	1.29
Utilities	17	1.00	17	0.86	17	0.82
Academic	1	0.84	1	2.35	1	0.51
			R2 > 10%			
Basic	4	1.27	9	1.21	12	1.12
Consumer Cyclicals	26	1.95	51	1.44	74	1.34
Consumer Non-Cyclicals	2	1.21	8	1.07	15	0.96
Energy	4	1.70	4	1.24	7	1.30
Financials	25	1.85	29	1.47	35	1.20
Healthcare	6	1.62	6	1.82	10	1.13
Industrials	25	1.89	45	1.42	66	1.34
Real Estates	1	1.83	3	1.22	9	1.12
Technology	17	1.96	28	1.51	45	1.41
Utilities	9	1.40	11	0.91	12	0.88
Academic			1	2.35		
			R2 > 20%			
Basic	1	1.61	6	1.28	9	1.21
Consumer Cyclicals	15	2.07	19	1.72	56	1.44
Consumer Non-Cyclicals			1	0.93	8	1.23
Energy	2	2.26			4	1.14
Financials	17	1.94	24	1.57	29	1.28
Healthcare	1	1.48	3	1.87	4	1.27
Industrials	12	2.11	17	1.67	48	1.47
Real Estates					5	1.13
Technology	7	2.04	6	1.49	26	1.51
Utilities	2	1.37	3	1.03	9	0.97
Academic						

the estimates. In fact, this would make it somewhat laborious to then determine the cost of capital for companies in this sector, and in turn make a valuation.

Again, the three graphs in Figure 3 illustrate the average βs calculated from daily (in red), weekly (in purple), and monthly (in blue) returns. The colors of the bars reflect the cutoff used to select the estimates: white bars identify the average βs obtained using all firms, light-colored bars correspond to a 10% cutoff, and dark-colored bars a 20% percent cutoff. The fourth graph shows the frequency distribution of the betas for each industry. Looking at Figure 3, the first clues that emerge are the divergencies between the average estimates calculated with the different cutoffs, which are more apparent when we apply a daily rather than a monthly frequency of returns. For example, recalling the data in Table 2 for the *Industrials* sector, which represents the second largest subsample (78 observations), the average βs from daily returns are, depending on the cutoff, 1.16 (full sample), 1.89 (10% cutoff) and 2.11 (20% cutoff), respectively; the differences are 0.73 (1.89-1.16) and 0.95 (2.11-1.16). On the other hand, using monthly returns for the same sector, the average βs are 1.23, 1.34 (10% cutoff), and 1.47 (20% cutoff), respectively, so the differences are somewhat less pronounced: 0.11 (1.34-1.23) and 0.24 (1.47-1.23). In general, with monthly data, the divergence using different cutoffs does not seem significant. For the *Real Estate* sector, it is almost zero (1.12 versus 1.13), while for the *Consumer Non-Cyclicals* sector, which appears to demonstrate the most marked variation, it still does not prove to be very significant (0.96 versus 1.23).

In general, if we analyze the averages in Table 2 with respect to, for example, returns calculated on a monthly basis, the most aggressive sectors for each cutoff level are (as we would logically expect) *Technology*, *Consumer Cyclicals*, and *Industrials*, with average βs of at least 1.2. The most defensive sectors

FIGURE 4. COMPARISON OF SECTOR AVERAGE βS, USING THE THREE DIFFERENT TIMEFRAMES AND DIFFERENT CUTOFFS
THE SIZE OF THE CIRCLES REPRESENTS THE AVERAGE SECTOR βS CALCULATED ON A DAILY BASIS



are, again predictably, *Utilities* and *Consumer Non-Cyclicals*. Overall, for the purposes of this study, clearly at the sector level, if the discriminant is whether the statistical quality of the estimates is used, this tends to have significantly more weight with higher frequency (daily) returns lower frequency (monthly) returns.

Figure 4 summarizes this underlying evidence and compares the sector average βs calculated on the three different timeframes (daily, weekly, and monthly) and for the different cutoff levels. According to industry, the x-axis shows the average β values calculated on a weekly basis, while the y-axis shows monthly calculations. The size of the circles represents the sector average βs calculated daily. The points should ideally lie along the bisector of the first quadrant, indicated by the dashed red line, if weekly data and monthly give similar results. With respect to daily data, this ideal situation should lead to circles with diameters growing gradually larger as we move to the right of the graph (i.e. as weekly average βs increase) and upward (i.e. as monthly average βs increase). Instead, the parameters generally lie above the bisector, indicating that monthly βs tend to overestimate weekly βs and in turn the cost of capital. In fact, the thicker blue line, representing the interpolation line of the points, always runs above the bisector. This phenomenon seems to intensify as the statistical quality of the estimate increases, i.e. from the first to the third graph in Figure 4. In fact, the bisector and the estimated line interpolating the points tend to diverge more and more as the quality of the estimate improves in the three graphs. In general, however, there seems to be more agreement between weekly estimates than monthly estimates, but much less agreement when looking at the daily data, where estimates seem to diverge quite significantly.

AN ATTEMPT TO ESTIMATE THE IMPACT ON THE VALUE OF COMPANY EQUITY

Based on the β estimates, next we attempted to track the effects of the different methodologies on the value of the company's equity. For this reason, and for simplicity's sake, we made several assumptions in an attempt to adopt solutions that were as illustrative as possible and focused only on aspects related to the methodology for determining βs. First, we determined the cost of equity capital from the various equity βs, applying the cost the Capital Asset Pricing Model (Sharpe, 1964; Lintner, 1965; Mossin, 1966), which relates the systematic risk measured by β to the return expected by shareholders investing the company's equity. The formulation and parameters in the analysis are as follows:

$$r_E = r_f + \beta_E \cdot premium = 3.73 + \beta_E \cdot 5.6$$

where r_f is the risk-free rate and *premium* is the market premium, that is, the premium that investors have historically demanded to invest in equities rather than risk-free securities. The 10-year BTP rate at the beginning of April 2023 of 3.73% and the historical market premium of 5.6% served as parameters for the two variables. The latter is the value obtained by Panetta and Violi (1999) in a well-known paper by the two former Bank of Italy researchers. βE is the previously calculated equity beta for each firm. To provide additional evidence, the return to shareholders was also computed using a prospective rather than a historical β following the approach of Blume (1975), which consists in adjusting the β coefficient by making it revert to the mean, which is generally considered to be equal to one. This adjustment is necessary because, as the author demonstrated, there is a systematic tendency for the extreme βs relative to a given period not to register outliers even in the subsequent period. The formal formulation of the adjustment is as follows:

$$\beta_{adjusted} = \beta_{historical} \cdot \frac{2}{3} + \beta_{prospective} \cdot \frac{1}{3}$$

where $\beta_{prospective}$ is assumed to be one. This method is also sometimes used in practice by appraisers. Finally, the weighted average cost of capital was determined for the different companies in the sample, using the data in the Eikon Thomson Reuters database when available as parameters. These are represented by the cost of financial debt, r_D, and the weights assigned to debt, w_D and equity, w_E, the latter being simply a complementary value to one of w_D. The formulation is as follows:

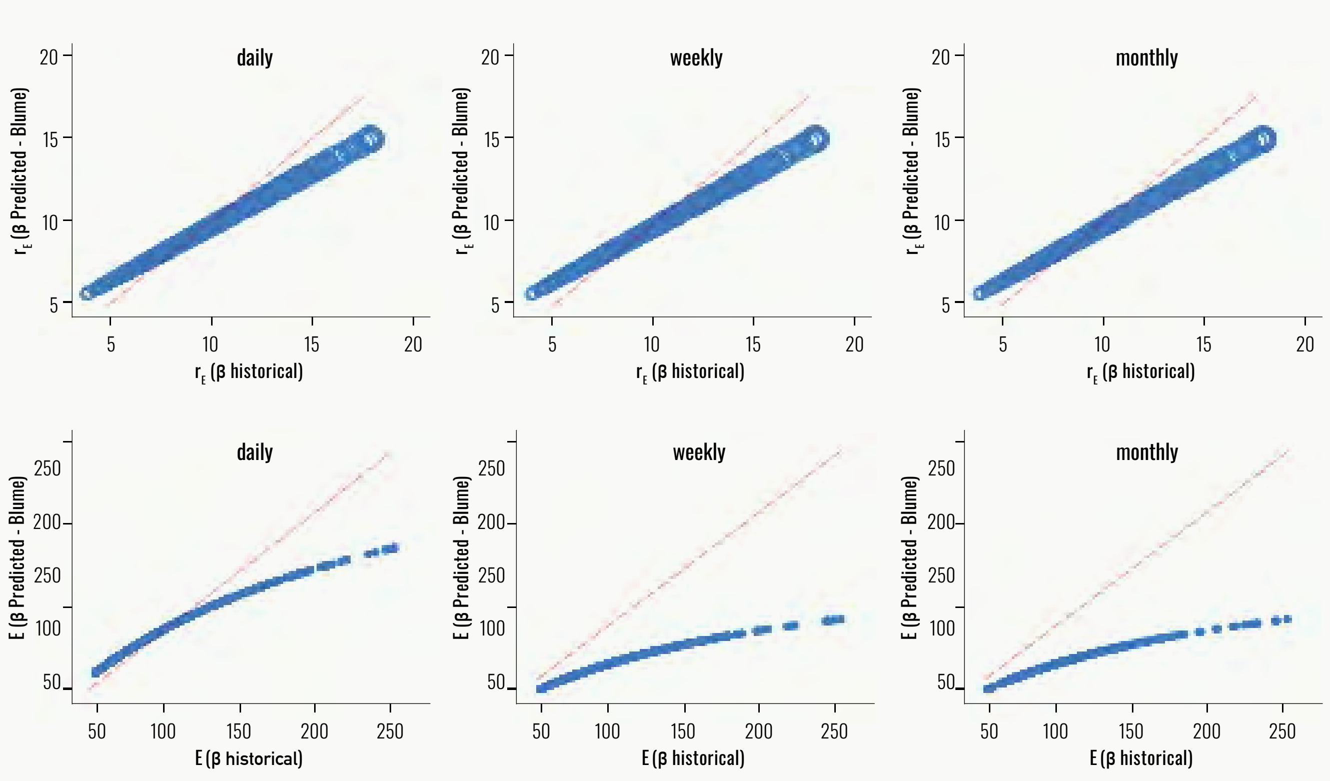
$$WACC = r_D \cdot (1 - \tau_c) \cdot w_D + r_E \cdot (1 - w_D)$$

where τ_c is the marginal corporate tax rate. The corporate income tax rate (IRES) served as this parameter because it is the only tax that allows full deductibility of borrowing costs, providing the tax shield on debt that reduces the cost of corporate debt. Finally, a company was assumed to produce a cash flow for shareholders of β10 million per year, a cash flow that is constant indefinitely, representing to some extent a business operating in a mature industry with extremely stable cash flows. It should be noted that this is quite a strong assumption, especially given the varying impact this choice could have on different sectors. At the same time, the idea is to capture the underlying aspects of how β is measured, rather than to draw a comparison across industries. In any case, with the Gordon and Shapiro (1956) model, the equity value of each firm in the sample was assessed as a function of the cost of equity capital previously estimated with the two different methods.

The results for equity returns and relative value are provided in Figure 5. The three graphs above illustrate daily, weekly, and monthly equity returns. The y-axis shows equity returns calculated using prospective β (Blume, 1975); the x-axis, historical β. The size of the dots, on the other hand, is a function of the WACC, which grows linearly as β increases. For example, as βs escalate and prospective rather than historical βs are used, the estimates of the cost of capital tend to rise. We can see this when comparing the cluster of points around the bisector of the first quadrant, which assumes that the two estimates (prospective and historical) lead to the

FIGURE 5. RETURNS (THREE UPPER GRAPHS) AND EQUITY VALUE (THREE LOWER GRAPHS)

The y-axis plots predicted βs (Blume, 1975); the x-axis, historical βs. In the first three graphs, the size of the points depends on the value of WACC. The red dashed line represents the bisector and accordingly indicates the values that result when the estimates obtained with predicted (Blume, 1975) and historical βs coincide.



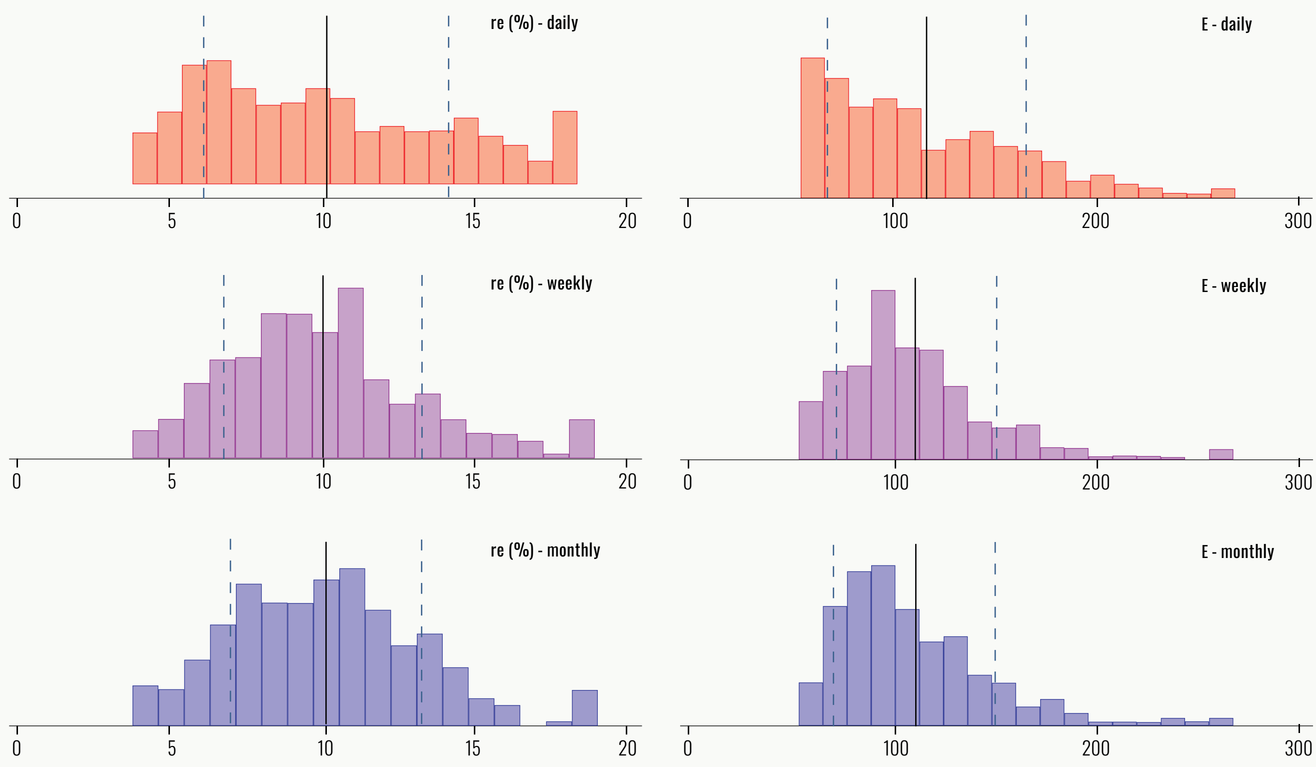
same result. In general, except for rather small β values, the two approaches generate lower estimates of the cost of equity, with an inverse effect on the value of the company's equity.

In the bottom three graphs in Figure 5, we focus on this specific aspect and instead represent the equity estimated using prospective β (y-axis) versus historical βs. Again, as the value of β rises, and with it business risk, the preference for one calculation method over another becomes relevant. As we would logically expect, with prospective βs, the riskier the firm's equity, the more conservative the estimates tend to become.

The aggregate distributions of returns obtained with the historical β estimates and the corresponding equity values are given in Figure 6, where the left graphs depict the distributions of the cost of equity capital using data from the three frequencies, i.e., daily, weekly, and monthly. The synthetic distribution values, represented in the graph by the vertical solid lines, are 10.14%, 10.00% and 10.10% respectively, so the values are quite similar. The graphs also show the $\mu \pm \sigma$ interval (blue dashed lines) to give an indication of the dispersion of the estimates. In this respect, as the frequency of the returns used to calculate β increases and the cost of equity capital rises, the latter tends to show more dispersed distributions, as a result of what we discussed above. The mean square deviations of the three distributions are 4%, 3.34% and 3.13%, respectively.

On the right side, again in Figure 6, the distributions of the equity value are shown instead. The mean values are €116 million, €111 million, and €110 million respectively, depending on whether daily, weekly, or monthly data are considered, and there is no reference cutoff for R^2. In other words, the values do not appear to be all that

FIGURE 6. DISTRIBUTION OF THE COST OF CAPITAL OF EQUITY (LEFT) AND EQUITY ITSELF (RIGHT) ALSO SHOWN ARE μ (SOLID LINE) AND RANGE μ ± σ (DASHED LINES)



different, at least based on a synthetic indicator of distribution such as the mean. In fact, the values are quite similar despite the fact that daily data provide less conservative estimates of stock values than weekly or monthly data. Again, as we would logically expect, using a daily frequency as opposed to a weekly or monthly frequency reduces the variability of the estimates. In fact, the standard deviation in the three cases goes from €49 million for daily to €39 million for weekly and monthly.

Finally, Table 3 shows equity values broken down by business sector. The top panel gives values based on daily estimates; the middle panel, weekly estimates; the bottom panel, monthly estimates. To make the table easier to read, histograms based on the values in the table are on the right. In particular, the column "Comparison based on R^2" measures up the estimates based on the previous three columns, i.e. the entire sample, the sample with βs that meet the requirement of R^2 greater than 10%, and finally those with R^2 greater than 20%. Clearly, the most conservative estimates are

TABLE 3. ESTIMATED AVERAGE EQUITY BY INDUSTRY

The top column is based on daily returns, the middle column is based on weekly returns, and the bottom column is based on monthly returns.
The last three columns show the daily-weekly-monthly (d-w-m) comparison.

	full value	R^2>10%	R^2>20%	comparison based on R^2	full d-w-m	R^2>10% d-w-m	R^2>20% d-w-m
Basic Materials	128.53	93.01	78.29				
Consumer Cyclicals	118.59	69.84	66.02				
Consumer Non-Cyclicals	141.88	98.52					
Energy	98.48	80.68	61.97				
Financials	115.00	73.57	69.95				
Healthcare	115.27	79.60	83.38				
Industrials	115.62	72.30	65.20				
Real Estates	138.76	71.52					
Technology	101.80	71.35	67.81				
Utilities	117.19	87.23	87.75				
Academic	118.87						
Basic Materials	113.58	95.74	92.19				
Consumer Cyclicals	106.21	88.38	78.05				
Consumer Non-Cyclicals	134.53	106.15	112.01				
Energy	122.61	98.86					
Financials	116.38	87.94	82.83				
Healthcare	136.00	76.55	74.10				
Industrials	106.33	89.96	78.44				
Real Estates	107.61	97.58					
Technology	103.46	87.07	83.65				
Utilities	121.18	115.11	105.12				
Academic	59.17	59.17					
Basic Materials	110.65	102.96	96.91				
Consumer Cyclicals	101.86	94.75	88.70				
Consumer Non-Cyclicals	133.43	120.69	106.07				
Energy	112.41	95.27	103.49				
Financials	122.08	99.58	94.48				
Healthcare	136.18	111.44	98.38				
Industrials	103.86	94.74	87.64				
Real Estates	109.07	107.42	104.20				
Technology	100.15	90.75	85.42				
Utilities	126.58	120.05	112.64				
Academic	151.28						

obtained using progressively more stringent cutoffs in all sectors. In the last three columns, however, the comparison concerns equity estimates using betas calculated at daily, weekly, and monthly (d-w-m) frequencies, depending on the cutoff. In general, it appears from this comparison that equity estimates tend to be more conservative with daily data as compared to weekly or monthly. For example, considering for the *Basic Materials* sector and a cutoff of 20% (last column), the equity value increases from €78 million for daily estimates to €92 million for weekly estimates and again to €97 million for monthly estimates. This clearly shows how the discriminant of the frequency of estimation decisively affects the final value of the company's equity. Specifically,, there is a reduction in equity value of about 20% when moving from monthly to daily estimates. Similar behavior is found for the *Consumer Cyclicals* sector. An overall reading of the values confirms the idea that the more we move toward more reliable estimates by responding to stricter cutoffs, the more we tend to underestimate firm equity. Instead with higher (daily) return frequencies for calculating β, we get progressively higher and less conservative equity estimates than what we would find with weekly or monthly frequencies.

CONCLUSIONS

The main objective of this paper is to provide updated evidence on the topic of empirical estimation of equity βs of firms operating in Italy, highlighting the differences in the values obtained depending on the methodological choices that the appraiser adopts. While the valuation of a company is, in general, an inherently complex task, in Italy this exercise appears even more problematic due to the often inadequate information practitioners are forced utilize. Moreover, the few empirical studies in the literature that have sporadically appeared mostly focus on contexts far removed from the Italian one, making the job even more difficult. Therefore, the present study examines the consequences on the value of the company's equity of two main methodological options for estimating equity from the practitioner's point of view. On the one hand, we can employ estimates of β with varying levels of econometrically reliable; on the other hand, we can apply various time frequencies (daily, weekly, monthly) for the calculation of equity returns to compute the parameter. In practice, there is no consensus on either choice. Our work has also shown the impact on value of using estimates based on prospective β (Blume, 1975) in contrast with purely historical β.

Therefore, this study has sought to highlight the implications of using one methodology over another. First and foremost, the analyses provide evidence on the estimates of β across sectors, stressing how these can vary considerably depending on methodological choices. At the same time, it is apparent that a more rigorous methodological strategy, while seemingly preferable, may unfortunately be hampered by the lack of available data. This means that such a methodology may not be fully applicable in some sectors. This being the case, our study also strived to underscore the impact on company value of adopting methodologies that, while potentially suboptimal, are used for lack of better alternatives.

In general, the significant differences between the results are influenced by the effects of different methodological choices. This fact accentuates how crucial it is to establish the valuation methodology to be adopted by the counterparties involved in a corporate transaction from the outset. This could be done, for example, at the time of acquisition, and should even encompass aspects that may seem unimportant. Through a detailed analysis of the issue at hand, this research provides an in-depth overview of the calculation of equity βs in business valuation and suggests interesting lines of research to pursue in the near future. Using estimates provided by well-known financial sites, such as *Yahoo Finance*, means running the risk of relying on black-box estimates. Although these are based on monthly observations over a three-year (36-month) horizon, they have the obvious limitation of using prices that are not adjusted for capital transactions to calculate returns. As if this were not enough, we must remember that the S&P500 market index, which consists almost exclusively of US large cap stocks, is used to estimate β.

Finally, it is important to point out that if we really want to study the peculiarities of the Italian context in terms of the cost of capital, an unquestionably fruitful line of research to explore in the near future is the cost of capital for non-diversified investors, who usually own and control the companies that typify the national context. Well-known authors (Kerins, Smith and Smith, 2004) assert that the cost of capital for such individuals could be anywhere from two to four times higher. Therefore, the hope is that future studies will delve into this aspect to enrich and broaden our understanding of the phenomenon. This would contribute developing effective research streams that would serve to assist entrepreneurs and practitioners in the vital and arduous task of estimating company value.

MANAGERIAL IMPACT FACTOR

- **Theoretical knowledge**: Managers need a solid understanding of corporate finance theories to effectively apply valuation methods and make informed decisions.
- **Contextual adjustment**: Risk factors such as equity beta (β) must be tailored to local market conditions, as reliance on foreign studies may misrepresent regional dynamics.
- **Data quality**: The accuracy of valuations depends on quality data. Managers must critically assess external sources to avoid misleading benchmarks.
- **Methodological balance**: Managers face a trade-off between rigor and practicality, often having to choose less-than-ideal methodologies due to data limitations while still producing reasonable estimates.

REFERENCES

Agrrawal, P., Gilbert, F. W., Harkins, J. (2022). "Time Dependence of CAPM Betas on the Choice of Interval Frequency and Return Timeframes: Is There an Optimum?" *Journal Risk Financial Management*, 15(11), 520.

Berk, J., De Marzo, P. (2013). *Corporate Finance*, Prentice Hall.

Blume, M. E. (1975). "Betas and their regression tendencies." *Journal of Finance*, 30(3), 785-795.

Corhay, A. (1992). "The intervalling effect bias in beta: A note." *Journal of Banking & Finance*, Volume 16, Issue 1, 61-73.

Dimson, E. (1979). "Risk measurement when shares are subject to infrequent trading." *Journal of Financial Economics*, 7, 197-226.

Fama, E. F., French, K. R. (1996). "Multifactor Explanations of Asset Pricing Anomalies." *Journal of Finance*, 1996, 55-84.

Fama, E. F., French, K. R. (1992). "The Cross-Section of Expected Stock Returns." *Journal of Finance* 47: 427.

Fama, E. F., French, K. R. (1993). "Common risk factors in the returns on stocks and bonds." *Journal of Financial Economics*, 3-56.

Gordon, M. J., Shapiro, E. (1956). "Capital equipment analysis: the required rate of profit." *Management science* 3, 102-110.

Graham, J., Harvey, J. (2001). "The theory and practice of corporate finance: evidence from the field." *Journal of Financial Economics* 61, 187-243.

Guatri, L., Bini, M. (2005). *Nuovo trattato sulla valutazione delle aziende*, Egea Editore.

Kerins, F., Smith, J. K., Smith, R. (2004). "Opportunity Cost of Capital for Venture Capital Investors and Entrepreneurs." *Journal of Financial and Quantitative Analysis*, 39(2), 385-405.

Lintner, J. (1965). "The Valuation of Risky Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budget." *Review of Economics and Statistics*, 47, 13-37.

Martikainen, T. (1991). "The impact of infrequent trading on betas based on daily, weekly and monthly return intervals: empirical evidence with Finnish data." *Finnish Economic Paper*, 4. Vol. 1, 52-64.

Mengoli, S., Sandri, S. (2012). "Decisioni finanziarie delle società quotate: un'indagine conoscitiva su obiettivi e metodi." In: (a cura di): GUIDA R. MELE A., Obiettivo crescita. p. 167-200, Bologna: Il Mulino.

Modigliani, F., Miller, M. (1958). "The Cost of Capital, Corporate Finance, and the Theory of Investment." *American Economic Review*, vol. 3, n. 48, pp. 261-297.

Modigliani, F., Miller, M. (1963). "Corporate Income Taxes and the Cost of Capital: A Correction." *American Economic Review*, vol. 3, n. 53, 1963, pp. 433-443.

Mossin, J. (1966). "Equilibrium in a Capital Asset Market." *Econometrica*, 34, 768-783.

Panetta, F., Violi, R. (1999). "C'è un Equity Premium Puzzle in Italia?" Banca D'Italia - Temi di discussione N. 353.

Scholes, M., Williams, J. (1977). "Estimating betas from nonsynchronous data." *Journal of Financial Economics* 5, 309-327.

Sharpe, W.F. (1964). "Capital Asset Prices: Theory of Market Equilibrium Under Conditions of Risk." *Journal of Finance* 19(3), 425-442.

Tirole, J. (2006). *The Theory of Corporate Finance*, Princeton University Press, 2006.

Treynor, J. (2005). "Why Market-Valuation-Indifferent Indexing Works." *Financial Analysts Journal* 61(5), 65–69.